STRENGTH OF MATERIAL

PROPERTIES OF MATERIALS

1. Axial force:

→ material को CIS मा normally लाग्ने

2. Bulk modulus

→ member is subjected to 3 mutually perpendicular stress.

→K = $\frac{direst\ stress}{volumetric\ strain} = \frac{P}{\frac{\rho v}{V}} = \frac{PV}{\rho v}$

3. Elastic curve

→ curve obtain from stress-strain relation

→ order found in curve is: proportional limit → Elastic limit → UVL→LYL→Ultimate Strength→ normal breaking stress→ actual breaking stress

→ Both yield point can be avoided by cold working

→ LYP is more significant than UYP due to less influence by shape of specimen.

→ yield point are not greatly influenced at ordinary temperature.

4. flexural strength (σ) :

→ also called as modulus of rupture, bend strength or transverse rupture strength.

→ stress before failure of material is called flexural strength.

→ Different value are

(i) $\sigma = \frac{F}{bd}$ (under axial axis)

(ii) $\sigma = \frac{3FL}{2bd^2}$ (3 point, lood is at cente)])

iii) $\sigma = \frac{FL}{4bd^2}$ (4 point, loading span(Li) = L/3)

(iv) $\sigma = \frac{3FL}{4bd^2}$ (4 point, Li other than L/2 or L/3)

5) Hook's low:

→ stress and strain are proportional to each other upto elastic limit.

→ Stress (σ)= modulus of elasticity (E) * strain (ε),

→ E is inversely proportional to temperature and 'Y' also inversely related.

6) Difference between modulus of elasticity and Young's modulus of elasticity

Modulus of elasticity (E)	Young's modulus of elasticity (Y)
E = 200 kN/mm² for steel and E = 220 KN/mm² for timber	Y = 275 KN/mm² for steel
E=$\frac{Stress}{Strain}$ upto elastic limit	$Y = \frac{Stress}{Strain}$ upto proportional limit

→ modulus of elasticity (E) has same unit of shear, pressure and modulus of rigidity

→ shear modulus of elasticity is also known as tangent modulus of elasticity.

shear modulus of elasticity = shearing stress / shearing strain

7) Property of Hook's low :

→Robert hooke introduce hooke's law.

→ glass obey's hook's law up to fracture.

→ if question says "limit within which hook's law hold good and if proportional limit is given in ans then tick PL otherwise EL.

8) Modulus of rigidity:

→ represented by G, N, or C

→ modulus of rigidity =$\frac{Shear\ Stress}{Shear\ Strain}$ upto proportional limit.

9) Poisson's ratio

→ Ratio of $\frac{Lateral\ strain}{Linear\ strain} = \frac{lateral\ strain}{longitudinal\ strain} = \frac{traverse\ strain}{axial\ strain}$

→ Poission's ratio is denoted by N or $\frac{1}{m}$

→ Different value of Poission's ratio are :

S.N	Description	Value
1	limiting value of poission's ratio	-1 and 0.5
2	value for steel a. for mild steel b. for high strength steel	0.25 – 0.33 0.25 0.33
3	Value for concrete	0.01-0.18≈0.2
4	value for ductile material	0.16-0.20

→ poission's ratio is always less than one

→Relation between poission's ratio, modulus of rigidity and bulk modulus is given by:

$$E = 2N(1+N) = 3K(1-2N) = \frac{9KN}{3K+N}$$

10) factor of safety (FOS):

→ FOS= $\frac{design\ load}{calculated\ load} = \frac{failure\ stress}{allowable\ stress} = \frac{ultimate\ load}{working\ load} = \frac{yield\ stress}{working\ stress}$

→FOS > 1 and in most cases = 2-3

→ FOS against overturning of retaining wall >= 2.

→ FOS against sliding of retaining wall >= 1.5

11) Ultimate stress

→ ratio of largest C/S area and largest load or force

→ the ratio of largest load to original C/S area is also ultimate stress.

12) Ultimate load

→ largest load which cause to break of material is called ultimate load.

→ the greatest load which spring can carry without getting permanently distorted is called proof load.

13) Brittleness

→ the property of material in which material break on impact of load.

→ Brittleness is opposite of ductility and compression test are carried.

→ Brittle material have following properties:

(i) they have low roughness due to small plastic deformation before fracture

(ii) they fails suddenly without warning.

14) Ductility

→ Ductility is defined as :

(i) Extension of material due to tensile force.

(ii) Elongation of material due to direct tensile test.

(iii) Reduction of material due to direct compression test.

→ Tensile test is used for ductility.

→ ductility is :

(i) directly proportional to percentage of area reduction

(ii) inversely proportional to carbon content.

Note:-

i. tensile आउने बित्तिकै ductility बुज्ने or tension आउने वितिकै

15) Malleability:

→ the ability of material to deform permanently under compression without breaking or fracture is called malleability.

→ material are changed to sheet and compressive force is used.

8) Strain:

→ deformation per unit length $=\frac{\partial l}{l}$

→ shear strain does not effect the size of member.

→Strain rosetters are used to measure linear strain:

→ total two incremental true strain = $\in_1+\in_2$

9) Stress

→ force per unit area (p) = F/A

→ unit is N/mm² or KN/m²

→ Load Transfer गर्न produce गरिने stress भनेको bearing stress हुन्छ

→ Under external load stress is developed in deformation of particle.

→ Stress due to temperature change depends upon supporting condition of end

→ Numerical value of tensile stress is more than theoretical or engineering stress.

→ Allowable stress in steel is less than yield stress.

→ maximum stress used in analysis of bending beyond elastic limit is yield stress

→ stress produced on elastic body does not depend on elastic constant

10) Shear center

→ beam को NA सँगको parallel line मा भएको point जसमा traverse force लगाउदा hoisting avoid गर्छ

→ point about which moment of shear flow is zero

→ point in plane of beam C/S through which resultant of shear force must pass

→ In F beam shear center lies at CG of section

→ shear flow means total shear stress of point

→ it has dimension of force per unit length.

→shear center is a point through which load must be applied to produce zero hoisting moment

→ In half circular section, shear center lies at $\frac{4r}{\pi}$

11) Different stress

→Stress induced due to temperature called thermal stress or temperature.

→Thermal stress is compressive if temperature increases and vice-versa.

→ temperature stress $= \propto \Delta t\ E$

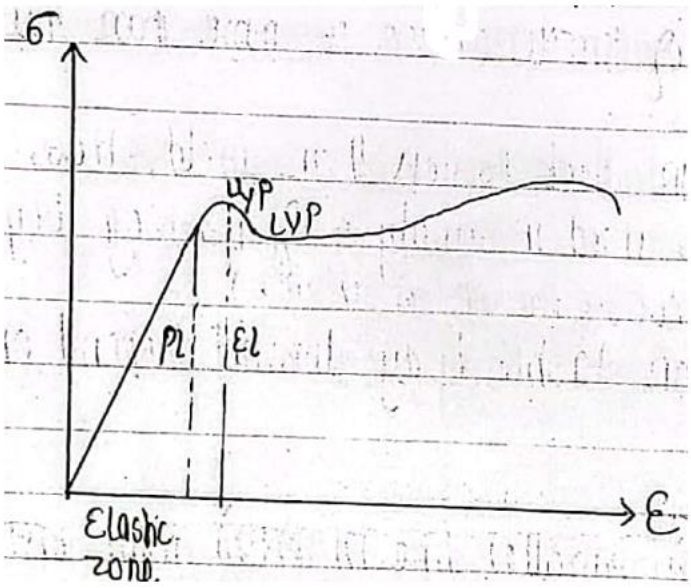
→ tapering section मा maximum stress smaller end मा हुन्छ

→ deformation stress means residual stress.

→ Crushing stress भनेको दुईवटा member को area contact भएर हुने

→ Temp is more in fixed beam compared to propped cantilevers.

Stress-strain diagram.



→ limit of proportionality depends upon type of material.

→ Engineering stress-strain curve and true stress-strain curve equal upto yield point.

→जब elastic limit पुग्छ तेस्पछी tensile strain increases more rapidly.

→ stress necessary to initiate yielding is more than that necessary to continue it

→ material लाई elastic limit beyond मा stress गरे permanent deformation occur हुन्छ त्यो चाहि material yield भएर हुन्छ load मा

→ Area under stress-strain curve of toughness दिन्छ

→ for homogeneous, isotropic & linearly elastic material minimum 2 constant are required to express stress-strain relationship.

Properties of material

Creep

→ the property under which material lends to deform continuously under constant or sustained wading is called creep.

→ Creep strain are caused by dead load only.

→ Creep causes the modular ratio to increase.

→ constant load or sustained छ भने creep मा tick लगाउने

→ creep = 3* instantaneous deformation.

→ creep coefficient :- 2.2 for 7 days.

1.6 for 28 days

1.1 for 1 year

→ Creep is also called as plastic deformation or plastic flow..

→Creep of material is time dependent strain of materials

17) Relaxation

→ strain in concrete at zero stress (constant stress) is called relaxation.

→ If strain decrease in constant stress than it is called relaxation.

→ If stress increase in constant strain than it is called creep loss.

→ if strain increase at constant stress than it is called creep loss.

18) Endurance limit

→ the property in which material doesn't fail in reversal of stress is called endurance limit

19) fatigue

→ the resistance of material decrease in reversal of stress is called fatigue

20) Hardness

→ property of material to resist abrasion.

→ Expressed by mohr's scale.

21) Stiffness:

→ the property of material to resist deformation under stress is called stiffness

→ the load required to produce unit deflection is also called stiffness.

22) Strength:

→ the property of material to resist force or deformation is called strength.

→ it is defined in the term of compressive strength, flexural strength, tensile strength etc..

Note: Option मा stiffness नभए strength मा tick लगाउने

23) Tenancy:

→ it is ultimate tensile strength of material or ultimate strength in tension.

24) Elasticity

→ the property by which material regain its original position after removal of load is called elasticity or elastic material.

→ the irrecoverable permanent deformation is called permanent set.

steel is highly elastic than rubber, brass etc.

25) Plasticity

→ it is opposite of elasticity.

→ Gold and lead are highly plastic material.

26) Elastic limit

→ the stress developed in material without permanent set is called Elastic limit

→ the material shows elastic behaviour up to elastic limit is called elastic material.

27) Resilience:

→ the energy absorbed or stored in material with in elastic limit. (i.e. without plastic deformation) is called resilience.

→ the maximum strain energy or greatest load stored in material within elastic limit is called proof resilience.

28) Toughness:

→ the property of material to deform permanently without breaking and having high strength is called roughness.

→ the property of material to absorb energy during plastic deformation. is defined as roughness.

→ the amount of energy stored up to fracture or plastic limit is called roughness.

→ the property of material which absorb energy at high stress (ie plastic deformation) without failure is called roughness.

→ the resistance of material to fracture by bending, twisting, fatigue or import of load is called toughness.

Resilience→ without plastic deformation.

toughness→ with plastic deformation.

Materials

1. Brittle material:

→ the material which fails without warning.

2. Homogeneous material:

→ Material having same property throughout volume.

3. Isotropic material:

→ Material having same property in all direction.

4. Orthographic material :

→ Material having different property in different direction.

5. Toughness material:

→ Material absorb energy at high stress without failure.

6. Visco-elastic material

→ Material which shows time dependent stress-strain relation.

→ In Different time show different stress strain relationship.

THEORY OF TORSION

Torque: → Twisting or turning force which tends to rotate material

→ it's unit is N/m or lb/ft

→Member subjected to twisting moment are subjected to shear stress.

Torsion:

→ It is defined as twisting of object due to torque. Unit = N-m, lb-ft

Torsion equation

$$\frac{I}{I_p \; or \; J} = \frac{\tau_{max}}{R} = \frac{C\varphi}{L}$$

T= torsional moment

R= radial distance

I_p or J = polar moment of inertia

φ= angle of twist

$C\varphi$ or $G\varphi$ or $N\varphi$ = torsional rigidity$= \frac{T}{\theta} = \frac{CJ}{l}$

→ torsional equation or formula is valid upto proportional limit.

→ torsional equation is valid for impact load and for circular section

Note about shaft

a. Hollow shaft resist more torque than solid. (having equal area)
b. Shaft subjected to twisting moment then every C/S of shaft will be in shear stress.
c. Shaft subjected to pure torsion crack, the crack will develop on periphery first then gradually towards center.
d. Shear stress or intensity will be zero at center to maximum at the circumference or top surface
e. Shear stress at a point is $\propto$ distance from axis
f. Shaft can resist maximum twisting moment = polar modulus * permissible shear stress
g. Horse power transmitted by shaft = $\frac{2\pi NT}{4500}$
h. Ratio of moment of resistance of solid circular shaft of dia. D and hollow saft (external dis D and internal d) =$\frac{D^4}{D^4-d^4}$
i. diameter of shaft =$\sqrt[3]{\frac{16T}{\pi\bar{C}}}$
j. Ratio of maximum bending stress to maximum shearing stress = $\frac{2T}{M}$
k. Ratio of wt of hollow soft to solid shaft =$\frac{1-n^2}{(1-n^4)^{\frac{2}{3}}}$ where n = ratio of internal and external dia of hollow shaft
l. Twisting moment is applied in plane of longitudinal axis.
m. Shear stress intensity is minimum at axis of shaft

Prismatic Questions

→ Elongation of conical bar is equal to 1/3rd of prismatic bar of same length

→ prismatic bar area will be infinite if stress produced by bar is equal to working stress.

→ prismatic bar having larger section module will be stronger in flexure.

Centroid and center of Gravity

CG was coined by Archimedes.

Material or C/S	C.G
1. Uniform Rod	Middle Point
2. Rectangle, square or parallelogram	Intersection point of diagonal
3. Triangle	Point where three medians meet
4. Semi Circular Arc	$\frac{2r}{\pi}$ above base
5. Semi Circle	$\frac{4r}{3\pi}$ above base and also above base of vertical face
6. Right Circular solid cone	h/4
7. Hollow Right Circular Cone	h/3
8. Solid hemisphere	3r/8
9. Hollow hemisphere	r/2
10. Solid Pyramid or solid Cone	$1/4^{th}$ of the total height above base
11. hollow pyramid or hollow cone	$1/3^{rd}$ of the total height above the base
12. parabola	$2/5^{th}$ of the axis OB above AC
13. Cube	L/2 from each face
14. Sphere	d/2 from every point
15. Trapezoid	$\frac{h}{3}(\frac{b+2a}{b+a}$

Moment of Inertia and Radius of Gyration

MOI→ second moment of area, unit is cm^4

→ first moment of inertia multiplied by perpendicular distance.

Radius of gyration:

→ denoted by K.

→ unit is cm

→K = $\sqrt{\frac{I}{A}}$

S.N	Section	Moment of Inertia
1.	Rectangular Rectangular lamina parallel to side ‘d’	$\frac{bd^3}{12} about\ C.G$ $\frac{bd^3}{3} about\ base$ $\frac{db^3}{12}$
2.	Square Section	$\frac{a^4}{12}$
3.	Solid Circular Section	$\frac{\pi D^4}{64}$
4.	Hollow Circular Section	$\frac{\pi(D^4 - d^4)}{64}$
5.	Semi Circular Section	$\frac{\pi D^4}{128}$
6.	Elliptical Section	$\frac{\pi \beta D^3}{64}$
7.	Triangular Section	$\frac{bh^3}{12} about\ base$ $\frac{bh^3}{36} about\ C.G$
8.	Polar moment of Inertia of shaft	$\frac{\pi d^4}{32}$
9.	Polar moment of Inertia of circular Section	$\frac{\pi R^4}{2}$

Bending Equation of beam in simple bending

Bending equation: $\frac{M}{I} = \frac{\sigma \text{ or } f}{y} = \frac{F}{R}$

Where, M= bending moment

I = moment of inertia.

σ = stress from NA

y = distance from NA to extreme fibre.

R= radius of curvature.

EI is also called flexural strength

section modulus $Z = \frac{I}{y}$

Note

→ plane section before twisting will remain plane after twisting if C/S is Circular.

→ In simple bending theory stress is proportional to strain at all section

→ beam subjected to pure bending, BM change. linearly and stress also

→bending formula le also called flexure formula which is valid for static load with no residual stress

→ product EI is called flexural rigidity or bending stiffness.

→ In this simple bending theory isotropic material elastic constant are same in all direction.

Assumption of simple bending theory

(i) plane section of beam remain plane after bending.

(ii) material obey's hook's law

iii)'E' is same for tension and compression

iv) beam bends to circular arc

v) load act perpendicular to beam axis.

Note:

→ plane section remain plane after bending means strain in fibers are proportional to their distance from neutral axis.

Mohr's Circle

→ Otto Mohr derived.

→ Along principal plane no shear stress act.

→ Principle plane are plane on which, resultant stress is normal stress

→ Extremities of any diameter of Mohr's circle represent normal stress on plane at 45°.

→ Maximum shear stress is given by radius of circle.

→ maximum shear stress is given by

→ Maximum normal stress at any point is a principal stress.

→ Minimum normal stress at any point is also principal stress.

→If Stress are equal than shape of mohr's circle will be a point.

→ volumetric strain in 3 direction is ($\varepsilon_X + \varepsilon_Y + \varepsilon_Z$).

Column Buckling

→ Theoritical buckling load are determined. Euler's formula

→ this formula is used for simply supported by pin joint i.e. pinned-pinned column

→Euler's bucking load $P_{cr} = \frac{\pi FI}{L_{eff}^2}$

Hinged at both end	Free-fixed	Fixed-fixed	Hinged-fixed
$L_{eff} = L$	$L_{eff} = 2L$	$L_{eff} = \frac{L}{2}$	$L_{eff} = \frac{L}{\sqrt{2}}$

→ Euler's formula hold's good for long column.

→ Euler's formula for column is used if λ >120 and λ >80

→Rankine - golden formula accounts for direct as well as buckling stress and is applicable to intermediate column.

→ length that satisfy Euler and Rankine-golden fomula is

$$L = \left(\frac{\pi EK}{fa - \pi^2 Ea}\right)^{\frac{1}{2}}$$

→ the critical stress on column for elastic bucking is given by.

Euler's formula = fc = $\frac{\pi^2 E}{\left(\frac{L}{r}\right)^2}$

Types of support

Roller Support:

→ permits movement in any direction in the plane of base

→ offer one reaction perpendicular to plane.

2. Hinged / Pin Support :

→ doesnot permit any movement at all but permit rotation about all three mutually perpendicular axes through joint

→ offer two reaction in two mutually perpendicular direction.

3. fixed support:

→ permit neither rotation nor translation along any of 3 principal axes.

→ offer 3 reactions.

4. free end :

→ permit rotation and displacement in any direction.

Types of Load

1. point load:

→ at the point of application of point or concentrated load there is sudden change of slope of B.M.

→ for point load s.f. diagram is st. line and B.M. diagram is inclined

2. UDL

→ for UDL, SF diagram is inclined and BM diagram is parabolic.

3. UVL

s.f. diagram is parabolic and B.M. diagram is cubic parabola

→ Inclined load generally causes thrust in beam.

Types of Beam

1) Simply supported beam :

(i) No. of point of contra flexure is zero for any type of loading.

(ii) If beam is subjected to various loading then Below table:

SN	load	Max. SF	Max. B.M
1)	UDL	$\frac{WL}{2}$	$\frac{WL^2}{8}$
2)	point load	$\frac{W}{2}$	$\frac{WL}{4}$
3)	UVL	$\frac{WL}{3}$ from right angle side and zero location is $\frac{L}{\sqrt{3}}$ from zero end ie (zero)	$\frac{WL^2}{9\sqrt{3}}$
4)	Couple at centre	$-\frac{C}{L}$	$\frac{C}{2}$
5)	UDL	reaction of prop $= \frac{5}{8}WL$	
6)	Gauss UVL with max. value at centre	$\frac{WL}{4}$	$\frac{WL^2}{12}$

2) Cantilever beam

(i) if UDL then sf = WL and BM= $\frac{wl^2}{2}$

(b) if UVL then BM = $\frac{wl^2}{6}$

(ii) if UDL, reaction of prop = $\frac{3wl}{8}$and reaction of fixed end = $\frac{5wl}{8}$

3 Propped Cantilever

→ Beam with one end fixed and other end simply supported.

→ If beam carries UDL then, Maximum positive B.M = $\frac{9}{12}Wl^2$at 3/8l

→If beam caries UDL then, then Maximum negative B.M = $\frac{16}{128}Wl^2$

→ point of contraflexure for UDL = 0.25l from fixed end.

4. fixed Beam

→ BM max for UDL=$\frac{wl^2}{8} - \frac{wl^2}{12} = \frac{wl^2}{24}$ with point of contraflexture at 0.211 $l$from both end.

→BM max for point load =$\frac{PL}{4} - \frac{PL}{8} = \frac{PL}{8}$ with point of contraflexure at 0.25$l$from both end.

→ Unknown element of reaction are horizontal and vertical component

5. Continuous Beam:

→continuous beam has equal span subjected to UDL throughout, the absolute maximum BM ocurs at middle of each span.

6. Overhanging Beam.

→point of contraflexure occurs in overhanging beam only (not in all other beam).

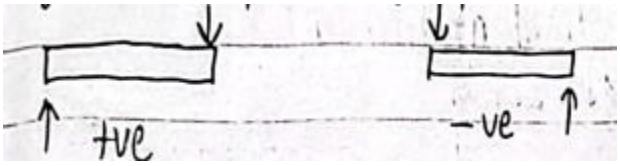
Axial, SF and B.M diagram

1. Axial force diagram

→Tensile positive and compression negative.

2. SF diagram

→left portion up and right portion down is taken positive and vice verca



→Difference between S.F value at be any two section is equal to area of loading between these two section

→ slope of curve of S.E diagram of any section is ordinate of Loading at that section.

→ SF = Σ transverse force either side of section

→ Shear force is generally associated with bending moments.

→ S.F for cantilever carrying UDL is triangle (right angled triangle).

3. B.M diagram

→ sagging is taken positive and hugging negative.

→BM= Σ moment of external load at both section.

→ Couple doesnot change shape of SFD

→ point where S.F is zero, BM is either minimum or maximum but येदी option मा दुईटै छ भने maximum मा tick लगाउने

→ BMD due to moment is always rectangle.

→difference between BM value at any two section will be equal to area of S.F between these two section.

→the slope of curve of B.M at any section will be equal to ordinate of S.F at that section.

BMD	Cantilever Beam
Rectangle Sq. Parabola Triangle Cubic Parabola	Free end subjected to moment UDL Concentrated load at free end Subjected

→ Beam subjected to pure bending moment means SF in length is zero

Members

1. Beam:

→Beam subjected to B.M. Strain & Stress is directly proportion to distance from NA

→Beam of uniform strength has same bending stress at every section.

→Rectangular beam Loaded transversly then compressive stress is developed on top fibre where as stress is developed on every Cross section.

→Rectangular beam loaded Longitudinally shear is developed on every horizontal plane.
→For beam of uniform strength, if depth is constant then B $\propto$ M.

→For beam of uniform strength if breadth is constant then D $\propto \sqrt{M}$.

→A flitched beam is used to equalize the strength in tension and compression.

→A beam is completely analyzed when shear and moments are found

→if uniform moment out on any beam then it's curve is parabolic arch.

→Beam of uniform strength are economical for large span.

2. Frames:

→ structure having moment resisting joint are called right frame.

→plane frame are 2-D structure conned together by rigid or hinged connection joint of frame are rigidly clamped.

→At either end of plane frame, maximum number of possible, transverse shear force is one and maximum no. of possible BM is zero

→In pinjointed frame force in member meeting at joint are coplanar. and concurrent.

→Frame is completely analyzed when variation in direct stress, shear and moment is found throughout frame.

Point of contraflexure

→It is also called point of vertical hinge or point of inflexion.

→it is point where B.M change sign or is either zero.

1. Deflection [y]

→transverse load को under मा beam को NA displace हुन्छ

2. Slope[θ]

→ Rate of change of deflection i.e. dy/dx

→ rotation & C/S of member or beam under traverse load.

→ In case of cantilever slope & deflection are maximum at free end and minimum at fixed end.

→ In case of simply supported slope is maximum but deflection is minimum at end.

→ In case of fixed beam both are zero at fixed end.

3. Curvature (1/R) :

→ Rate of change of slope i.e. $\frac{d\theta}{dx}$

→Axis of beam bends in curve which is called elastic curve or deflection curve

Here,

$i) \frac{1}{R} = \frac{d\theta}{dx} = \frac{M_x}{EI}$

$ii) \frac{dy}{dx} = \frac{\theta_x}{EI}, first\ derivative\ gives\ slope$

$iii) \frac{d^2y}{dx^2} = \frac{M_x}{EI}, second\ derivative\ gives\ B.M$

$iv) \frac{d^3y}{dx^3} = \frac{dM_x}{dx\ EI}, third\ derivative\ gives\ S.F$

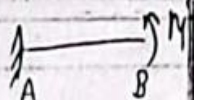
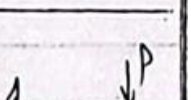
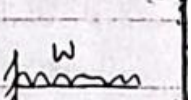
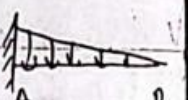
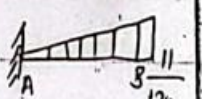
$v)\ \frac{d^4y}{dx^4} = \frac{dv}{dx}.\frac{1}{EI} = \frac{W_x}{EI}\ , fourth\ derivative\ gives\ load\ intensity$

Free body diagram

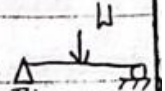
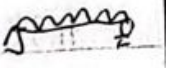
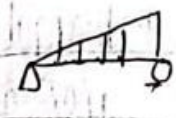
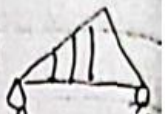
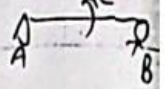
→ the diagram of body or part of body in isolated equilibrium

→ FBD will be in equilibrium under action of all super imposed load, self-weight, support reaction and internal reaction.

SF and BMD of Determinate Structure

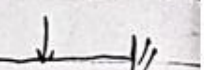
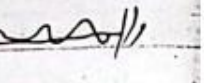
N	Nature of load	Member	SFD	BMD	Max deflection	Slope	figure
		Cantilever beam					
1.	moment at free end	cantilever	St. line i.e no shear force at all	Rectangle	$y_B = \frac{ML^2}{2EI}$	$\theta_B = \frac{ML}{EI}$	
2.	point load at the end	cantilever	Rectangle	triangle	$y_B = \frac{WL^3}{3EI}$	$\theta_B = \frac{WL^2}{2EI}$	
3.	UDL	cantilever	triangle	sq. parabola	$y_B = \frac{WL^4}{8EI}$	$\theta_B = \frac{WL^3}{6EI}$	
4.	UVL with max. at fixed support	cantilever	sq. parabola	cubic parabola	$y_B = \frac{WL^4}{30EI}$	$\theta_B = \frac{WL^3}{24EI}$	
5.	UVL with max. at free end	cantilever	sq. Parabola	cubic parabola	$y_B = \frac{11WL^4}{120EI}$	$\theta_B = \frac{WL^3}{8EI}$	

Simply supported

1.	point load at center	S.S beam	rectangle	triangle	$y_c = \frac{Wl^3}{48EI}$	$\theta_c = \frac{Wl^2}{16EI}$	
2.	UDL	S.S beam	triangle	sq. parabola	$y_c = \frac{5Wl^4}{384EI}$	$\theta_A = \frac{Wl^3}{24EI}$	
3	UVL (zero at one end and max. at other)	S.S beam	triangle	parabola	$y = \frac{0.0065Wl^4}{EI}$	$\theta_A = \frac{7Wl^3}{360EI}$ $\theta_B = \frac{-Wl^3}{45EI}$	
4.	triangular load max. at center	sq parabola	cubic parabola		$y = \frac{Wl^2}{60EI}$	$\theta = \theta_t$	
5.	Couple at center	st. line	rectangle		$y = \frac{Ml^2}{36\sqrt{3}EI}$	$\theta = \frac{Ml}{24EI}$	

※ fixed beam ※

(FEM) → गति आघूर्ण

	BM (max)	deflection	
1. point load at centre	$\frac{Wl}{8}$	$\frac{Wl^3}{192EI}$	
2. UDL	$\frac{Wl^2}{12}$ and $\frac{Wl^2}{24}$ (centre)	$\frac{Wl^4}{384EI}$	
3. triangular load (zero at one end max at other)	$\frac{Wl^2}{30}$, $\frac{Wl^2}{20}$	$\frac{5Wl^4}{96EI}$, $\frac{Wl^4}{32EI}$ (centre)	
triangular load max. at centre	$0.00131\,Wl^2$	$\frac{0.73Wl^4}{384EI}$	

Types of structure.

1. Determinate structure

→Analyzed using equilibrium equation or equation of statics

→E.g:- simply supported beam, cantilever beam, single and double overhanging beams , 3-hinged arch. etc.

→ Equation of static

2. Indeterminate or redundant structure:

→Analyzed using equation of static and necessary number of equation of compatibility.
→ Internally indeterminate structure may be unstable structure based on support
e.g:2 -hinged arch, fixed arch, multi-storied frame etc.

→ Externally indeterminate structure is one in which external reaction component are more than equation of statics

→ equation of compatibility are written based on the geometry of deformed structure under the action of several force acting.

Plane structure

→ Analysis of structure is calculating the magnitude and nature of various straining action at silent points of structure.

→Design of structure is deciding material and proportion of various member of structure

→ plane structure is structure in which various member lie in a plane.

→1-D structure is one in which one dimension is larger than other 2 dimension.

→ 2-D structure is one in which two dimension are larger than third.

→plane structure subjected to force lying outside plane will be unstable.

Prop

→It is an additional support provided to a stable structure to avoid any displacement at desired point in direction perpendicular to plane of prop.

→A rigid prop is one which does not permit any displacement perpendicular to that plane of prop.

→ An elastic prop is one which develop reaction proportional to compression in itself.

→A sinking prop is one, which is provided below regular support level and become effective after respective deflection occurs.

Analysation of determinate and Indeterminate structure.

(A) Truss	(B) Frames and Beam
1. 2D/ plane truss/ simple truss	1. 2D/plane frame and beam
a. m+r <2j determinate but unstable	a. (3m+r)< (3j+c) unstable or mechanism
b.m+r =2j statically determinate	b. (3m+r) =(3j+c) determinate
c. m+r >2j statically indeterminate	c. (3m+r)>(3j +c) indeterminate
2. 3D/ Spaced Truss	2. 3D/ Spaced Frame
a. m+r <3j determinate and unstable	a. (6m+r)< (6j+c) unstable
b.m+r =3j determinate	b. (6m+r) =(6j+c) determinate
c. m+r >3j indeterminate	c. (6m+r)>(6j +c) indeterminate

Truss

→Total degree of redundancy in truss (n) = (mtr) -2j

→Total external redundancy in truss (E) = R-3

→Total internal redundancy in truss. (I)= n-E

Frame

→Frame for hinge in 2D C = Σ (m-1)-m

→ for hinge in 3D frame C= 3 (Σm-1)

→ total degree of indeterminacy (D_s or n) = (3m + r)-(3j + c.).

→ total External redundancy (D_{se}) = R-3.

→ Total internal redundancy (D_{si})= D_s + D_{se}

Simple method:

(A) If rigid frame then $D_{si} = 3C\text{-}S$ (S=0), c = dose loop

$$D_{se} = R\text{-}3$$

(B) If hinge internal than $D_{si} = 3C - S = \Sigma (m_i - 1)$

m_i = members attached to one hinge.

Determinate/Inderteminate	Degree os static indeterminacy
if all reaction are parallel then structure is unstable. i.e. all are rollar support.	DSI=no. of unknown- equation of statics =R - 3
a. m< 2j-3 Determinate but unstable	a. DSI<0 unstable
b. m=2j-3 Determinate	b. DSI=0 stable and determinate
c. m>2j-3 indeterminate	c. DSI>0 stable and indeterminate

Degree of Kinematic Indeterminacy

The number of joint translation and rotation is called degree of kinetics indeterminacy. It is also called degree of freedom. A structure without degree of freedom is called Kinematically determinate. DKI for Structure is taken as 2.

DK1 = 3j-r for rigid structure (frame, beam)

= 2j-r for pin jointed structure (truss)

the total number of joint displacement is called DKI.

Method to find slope, and deflection

1) Double Integration method.

2) Macauly's method

3) Moment area method

4) Conjugate beam method

(5) Castigliaon's theorem

(6) Strain energy method,

7) Superposition method

8) unit load method

Force or Compatibility Method or flexibility method

→ redundant are taken as unknown where redunder are force.

→ Common methods are

1. Flexibility coefficient, consistent deformation method
2. Conjugate beam method
3. cashgliano's theorem
4. Column analogy method
5. Maxwell-mohr equation
6. Method of least work
7. Three moment equation
8. Elastic center method
9. Unit load method

Equilibrium or deformation or stiffness coefficient or displacement Method or matrix method

→ Displacement of joint are taken as unknown.

Methods are:

1) slope deflection method.

2) Moment distribution method.

3) Minimum potential Energy method

4) Icani's method / rotation contribution.

→ Stiffness method of structural analysis is also called unit load method.

Energy Method

(1) strain energy or real work method

2) Maxwell reciprocal theorem.

(3) Betti's theorem

(4) Castigliano's theorem

Analysis of determinate structure

No. of methods are available and some of them are:

सबै method linear elastic body मा apply हुन्छ

1. strain energy or real work method

   कुनै पनि structural member external load मा imposed छ भने तेस्ले energy store गर्छ जस्लाई strain energy भनिन्छ

   → complementary strain energy भनेको nonlinear elasic body मा store हुने strain energy हो

   → linear elastic body मा चाहिं strain energy हुन्छ

   →strain energy store per unit volume is called modulus of resistance and equal to 0.5*stress*strain

   → strain energy = 0.5*stress*strain*volume.

   Strain energy Shored = work done by external load ($\sum_0^n \frac{1}{2} p * s$)

   →It is suitable for structure subjected to single concentrated load and deflection of at a point.

   → In case of rigid body no strain energy because no real workdone.

   →the work done per unit volume when tensile force gradually increases from zero to value causing rupture is called modulus of toughness.

   ➔the amount of strain energy stored within elastic limit i.e. without plastic deformation is called resilence.

   ➔The amount of strain energy store within elastic and plastic limit i.e. upto fracture is called toughness.

   ➔The maximum amount of strain energy stored within elastic limit is called proof resilence

   ➔Stress developed in material without any permanent set is called elastic limit.

   ➔As compared to axially loaded member Strain energy stored in bending is 1/3

   Formulas

   (1) SE due to axial force = $\frac{p^2 L}{2AE}$

(2) SE due to BM = $\int \frac{M^2 dx}{2EI}$

(3) SF due to torsion = $\frac{T^2 L}{2GJ}$

(4) SF due to shear = $\frac{p^2 L}{2AG}$ or $\int \frac{p^2 dS}{2AG}$

2. Virtual work method:

   known as unit load or dummy load method.

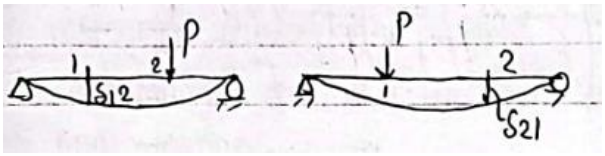
   developed by Johanm Bernoulis

   ➔virtual force * real displacement = virtual internal force * real internal displacement

   → this method results in equilibrium of forces.

3. Maxwell's Reciprocal theorem :

   $\delta_{12} = \delta_{21}$



→ applied for linearly elastic body.

4. Betti's theorem :
   → applied for linearly elastic body.
   →$P_1 * S_{21} = P_2 * S_{12}$
5. Castigliano's theorem :

   → applied for linearly elastic body.

   $\frac{\partial U}{\partial P} = \Delta$

   where

   U= strain energy.

   Δ=Displacement

   P=load

→Castigliano's first theorem is applied only when principal of superposition is valid and second theorem is applied to any type of structure to find deflection.

Analysis of indeterminate structure's

Two methods are used

1. Compatibility method
2. Equilibrium method.

1. Three moment theorem

   →developed by clapeyron

   → used for fixed and continuous beam

   → used in no die continuity such as hinge

   → Equation is: $\frac{M_1L_1}{EI_1} + 2M_2\left(\frac{L_1}{EI_1} + \frac{L_2}{EI_2}\right) + \frac{M_3L_2}{EI_2}$

   $$= -\frac{6A_1X_1}{EI_1L_1} - \frac{6A_2X_2}{EI_2L_2}$$

   → fixed end of continuous beam is changed by adding span of zero length.

2. Column Analogy method :
   → developed by hardy cross
   → applied to structure having degree of indeterminacy less than 3.
   → Suitable for finding fixed and moment and carry over factor for non-prismatic members.

3. Slope-deflection method:

   →Proposed by G.A maney and developed by Axel bendixen.

   → slope means rotation of joint and deflection means translation of joint.

   → used to find S.F and B.M. where slope and deflection are unknown.

   →Deformation are caused by B.M only (PSC)

   Equation is

   $$M_{AB} = \overline{m_{AB}} + \frac{4EI\theta_A}{L} + \frac{2EI\theta_B}{L} - \frac{6EIS}{L^2}$$

$$M_{BA} = \overline{m_{BA}} + \frac{4EI\theta_B}{L} + \frac{2EI\theta_A}{L} - \frac{6EIS}{L^2}$$

→joint equilibrium i.e $M_{AB} + M_{BA} = 0$ is used when no moment at joint.

→ simultaneous equation of scope deflection can be solved by iteration in momoment distribution method.

→ the end moment for any member are expressed in term of the unknown end rotation.

4. Moment distribution method:

→ developed by hardy cross and is best suited for trussed beam.

→Moment distribution method is displacement method.

→ Moment distribution method is iterative method and approximate method.

→ All joints in frame are rigidly clamped.

➔Stiffness is defined as force / moment to cause unit displacement.

➔Axial Stiffness → axial force to cause unit deflechon.

➔flexural / bending stiffness → moment to cause unit rotation

➔torsional stiffness → torque to cause unit twist.

➔Absolute stiffness:

→ far end hinged वा fixed हुदा , simply support मा unit rotation produce गर्न चाहिने moment

→ when far end is fixed, K = $\frac{4EI}{L}$

→ when far end is hinged, K = 3EI/L

➔ Carry over factor :

Ratio of moment induced at far end to moment applied at near end.

when far end is fixed, CO = 0.5 (also for prismatic member-0.5)

when far end is hinged, CO = 0

➔ Distribution factor :

Ratio of stiffness of that member to total stiffness of member meeting at joint i.e.

Df = K/ ΣK=Moment/ ΣMoment

i) At fixed end, Df= 0

ii) At hinged end, Df = 1

iii) sum of Df of all member meeting at a joint is 1.

5. Unit Load Method :

→It is also called Maxwell method

→Used in truss to find deflection.

Deflection in pin-jointed truss

1) Unit load method

2) Castiglion's method

3) Angle weight method

11 joint displacement method

5) Willot - Mohr's method

5) Stiffness matrix method

→ Unit load and castigliono's are used to find deflection of single joint at a time.

→ pin jointed frame are subjected to angle weight and joint displacement method are used to find deflection of all joint at a time.

→ Stiffness method is used to find deflection and force in member.

→ Willot mohr diagram re used to find displacement of joint and is graphical method.

→ For finding force in member of truss by the graphical method, the force diagram was suggested by maxwell.

→ A compound truss may be formed by connecting two simple rigid frame and by three bars.

→ If m_1, and m_2 are two individual simple truss then compound truss will be rigid and determinate if $m=m_1 + m_2 +3$

→Force in truss can be analyzed by method of joints, section and graphically.

In pin jointed frame all loads are assumed to act in the plane of frame and at the joint of members only

Portal method of approximate analysis

→ used when height is less than width i.e. for low rise building

→ inflection point is located at mid height of each column and centre of each beam.

→ the frame which are casted monolithically are called portal frame.

Conjugate Beam method

→ Developed by Muller-Breslau

→ theorem 1: slope at point in real beam = shear of conjugate

→ theorem 2: displacement in real beam = moment of conjugate beam

→ load on conjugate beam = M/EI diagram on actual beam

Real Beam	Conjugate Beam
Simple Supported	Simple supported
Fixed End	Free end
Free end	Fixed End
Span को बिचमा internal hinge भए	Rollar Support
point of zero slope	Point of zero shear
point of max. deflection	Point of max. moment
Length (L) of span	Length(L) is same

Influence line diagram

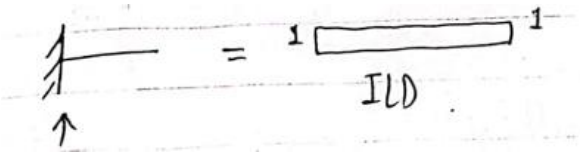
ID is diagram due to unit moving load on structure usually rom left to right.

ID can be drawn for both determinate and indeterminate structure i.e. for any type structure.

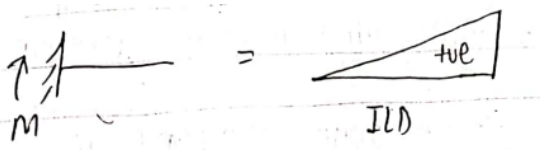
Area of ILD for moment at fixed end (A) = $L^2/16$ = ½ * L * 1/8

1. Cantilever

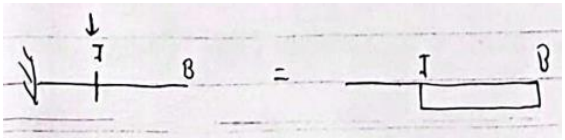
ILD for reaction at support = rectangle with unit ordinate



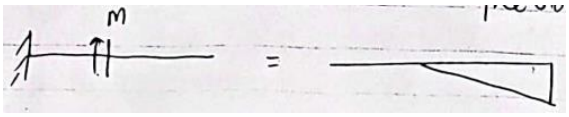
ILD for moment at support = triangle with max ordinate at free end.



ILD for SF al any section = Rectangle with unit ordinate between free end and the section.

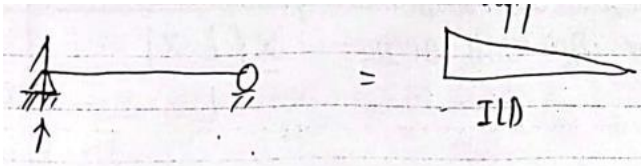


ILD for moment at any section= triangle between section and free end with ordinate max at free end

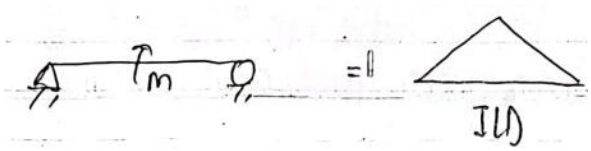


2. Simply supported:

ILD for reaction at support = triangle with max ordinate at that support.



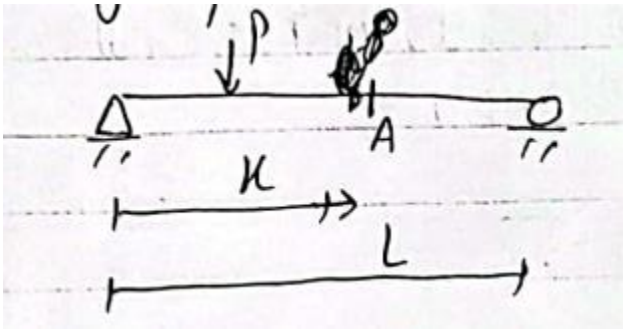
ILD for moment at any section = triangle with maximum ordinate at section and zero at Support



Arch

ILD for arch = B.M.D - Hy at any section.

Single point load



max -ve sf occur when load is just left of section (x/L).

max +ve sf of occur when load is just right of section (L-x)/L

Max BM will occur when load is on the required section.

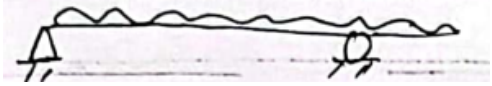
i.e. under the load max. BM will occur. $\frac{x(L-x)}{L}$

absolute may -ve sf occur when load is on right end and equal to 1.

absolute max. +ve sf occur when load is on left end and equal to 1.

absolute maximum moment occur when load is at mid span and equal to (L/4)

UDL longer than span :



1. Load intensity and area of ILD and products give value of sf and Bm
2. Max (-ve) sf occur when UDL is just left of required section.
3. Max (+ve) sf occur when UDL is just right of required section.
4. Absolute max (-ve) sf occur when UDL cover entire span and at right end and equal to 0.5
5. Absolute max (-ve) sf occur when UDL is on left end and cover entire span and equal to 0.5.
6. Absolute max. BM occur when UDL cover entire span and at mid.

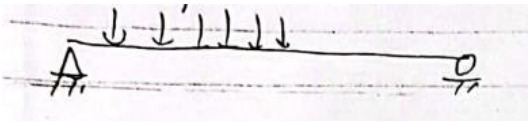
UDL is shorter than span :

→ Max (-ve) sf occur when head of load touches the section.

→Max (+ve) sf occur when tail of load touches the section.

→Max. BM occur when load is at section and section divides the load in same ratio.
→Absolute max. BM occur at mid span when C.G of load lies at that mid span.

Train of concentrated load



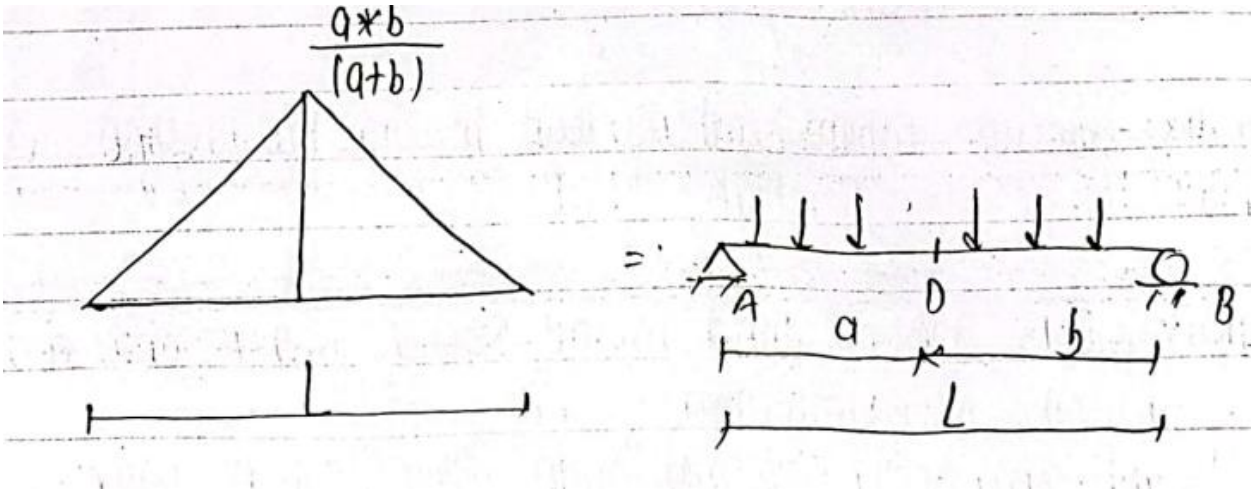
→Max (+ve) or (-ve) sf occur at section when one of load is at the considered section and calculated by trials.

Absolute max (-ve) sf occur when one load lies in left support and calculated by trials.

Absolute max (+ve) sf occur when one load lies in right support and calculated by trials.

Max. BM occur when avg load is on section and at center of beam and resultant of load.

Max absolute BM occurs under one of load not between load.



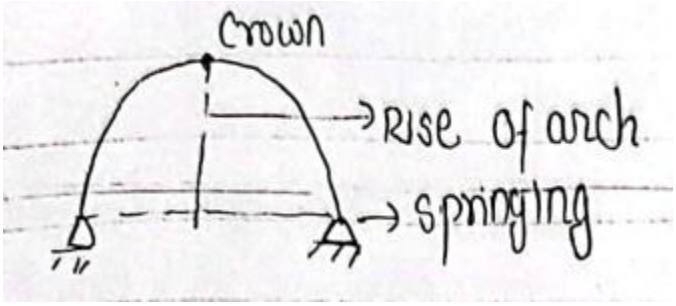
→Avg load on AD= avg load on BD

Note:

→ILD for SF at a section, the sum of maximum -ve, and maximum +ve ordinate is 1.
→Ordinate of ILD for bending moment has dimension of length.

→In plane truss influence line value varies linearly between panel points.

ARCH



A. TYPES OF ARCH

1) Tied arch

2) Hingeless arch

→ it is of 2nd degree

3) two-hinged arch

4) three-hinged arch

3) Two-hinged arch:

→ it is of one degree.

→it is statically indeterminate

→ Horizontal thrust (H) subjected to UDI (H) =$\frac{WL^2}{8h}$

→ horizontal thrust for UVL (H) = $\frac{WL^2}{16h}$

→ for 2- hinged parabolic arch if one support settle horizontally then horizontal thrust decreases but it settle vertically horizontally thrust remain unchanged.

4) three-hinged arch

→ it consist of two plate and is statically determinate.

→ Horizontal thrust for UDL (H) = $\frac{WL^2}{8h}$

→ linear and actual arch meet at least at three points.

→ In 3-hinged arch, SF is usually maximum at quarter point.

B. OTHER FORMS OF ARCH

1. parabolic arch :

→equation is $y = \frac{4hx(1-x)}{L^2}$

→ rise at quarter span = 3/4th of rise of crown

→if crown is origin then equation becomes $\frac{x^2}{y} = a, a = \frac{L^2}{4h}$

→ In parabolic arch BM and SF are zero.

→ 3-hinged parabolic arch subjected to UDL every section resist compressive force.

→ 3-hinged parabolic arch subjected to point load, ILD will be triangle with maximum ordinate of center.

2. Circular arch

→ Radius (R) = $\frac{L^2}{8h} + \frac{h}{2}; R = rise\ of\ arch$

→ coordinate at any point are x= $\frac{L}{2} - R.\sin\theta\ \ and\ y = h - R(1 - \cos\theta)$ from left support.

→for 2-hinged semi-circular arch subjected to UDL (H) = $\frac{4}{3}\frac{WR}{\pi}$

→ for 2-hinged arch subjected to UVL in circular arch (H)= $\frac{2}{3}\frac{WR}{\pi}$ this value also can be used for right half span.

→ for 2-hinged semi-circular arch, locus of reaction is straight line.

C. Normal thrust and Radial thrust

→ normal thrust (N) = $U\sin\theta + H\cos\theta$

→ Radial thrust (Q) = $V\cos\theta - H\sin\theta$

→ normal thrust is 10 times more than radial thrust

D. PROPERTIES OF ARCH

→ Arch is subjected to thrust, SF and B.M

→linear arch represent thrust line and masonry arch is subjected to thrust only

→ line of thrust of linear arch is Imaginary line of thrust for given set of load.

→ In solid arch, the sf at act perpendicular to axis of arch.

→ If axis of arch coincide with line of thrust then B.M is zero

→ Effect of arching of beam is to reduce B.M throughout.

→Eddy's theorem is valid for vertical load.

E. Tricky QUESTIONS

(1) In 3-hinged arch the third hinge is generally kept at

a) crown of arch

b) midpoint of crown and left support

c) midpoint of crown and right support

d) None of these.

2) A three hinged arch is generally hinged at its support and

a) at one quarter span

b) at the crown

c) anywhere in the rib

d) none of these

3) A three hinged arch will have 3 hinges.

a) two at the end and one anywhere in between 2 ends.

b) two at the two end and 3rd at crown only.

c) one hinge at the crown and other at anywhere

d) none of above

Cable

→ cable is subjected to direct tension and are economical for large span.

→ cable supporting load will be under tension only.

→ cable subjected to UDL assume shape of parabola.

→ maximum tension in cable occurs at highest point.

→ B.M in cable is zero at all point carrying load.

→ stress in cable is inversely proportional to temperature and length.

→length of cable (L) = $l + \frac{8}{3}\frac{y_c^2}{l}$; $l = span\ of\ cable\ and\ y_c = maximum\ sag$

→Horizontal thrust (H)=$\frac{Wl^2}{8y_c}$

→ Horizontal reaction on cable is constant at any point.

PROPERTIES OF NEUTRAL AXIS

→Along N-A in simply supported beam fibre do not undergo strain

→N-A of beam C/S should pan through centroid of the the section

→At N-A of Beam:

1. Shear stress is maximum
2. Bending stress is minimum.

→In beam neutral plane doesnot change during deformation.

→Concrete below N-A embades tensile steel only

→NA of T-beam lies with in :

1. flange
2. below the slab.
3. and concide with geometric centre of beam

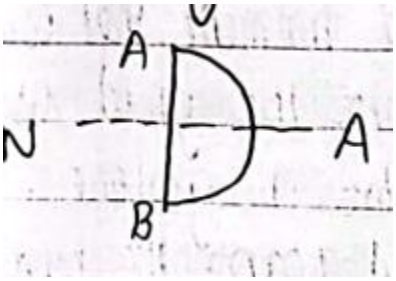
T-beam behaves as rectangular beam if NA lies within flange.

Shear stress

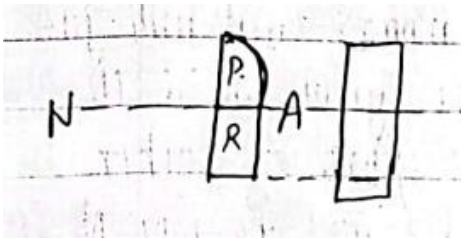
→ Shear stress in homogeneous beam is parabolic with maximum at NA : (same for W.S.M and L.S.M)

→Shear stress in R. C beam increases from zero at the top face of the beam parabolically to maximum at NA and then uniform or rectangle to C.G. reinforcing bar. (same for W.S.M and L.S.M).

Homogeneous beam



R.C beam.



Ratios

→ shear shess $= \frac{SF}{LA\ X\ breadth\ of\ beam\ or\ perimeter}$; $LA = lever\ arm$

→ $\frac{Max.shear\ stress}{Avg\ shear\ stress} In\ rectangular\ beam\ =\ 1.5$

→ $\frac{Max.shear\ stress}{Avg\ shear\ stress} In\ triangular\ beam\ =\ 1.5$

→ $\frac{Max.shear\ stress}{Avg\ shear\ stress} In\ circular\ beam\ = \frac{4}{3}$

→Due to shear stress, the material may fail by both compression (Crushing) and tensile (splitting) stress

Plastic analysis

→ Main principal of plastic analysis is based on distribution of stress at certain in structure reach the field point

i.e. Plastic state पुग्नु भनेकै yield वा fail हुनु हो।

plastic hinge in a section is caused when all fibres at a section reach plastic state.

Yield moment	Plastic moment
It to the moment that will just produce the yield stress in outermost fibre of section. In loaded beam, the moment at which the first yield occurs is called yield moment	→ It is the moment that will just produce the yield stress in the fibre everywhere. → In loaded beam the moment at which entire section of beam become fully plastic is called plastic moment → it creates the plastic hinge..

Plastic hinge

At the plastic hinge location the radius of curvature is infinite segment between two successive plastic hinge deform in shape of rigid material.

Plastic section modulus

The ratio of plastic moment capacity of section to yield moment.

It can be obtained by taking static moment of tensile and Compression areas about NA. The ratio of plastic section modulus to elastic section modulus is always greater than one.

Shape factor:

Given by:

$$\frac{Plastic\ Moment}{Yield\ Moment} = \frac{M_p}{M_y}$$

Shape/ C|S of section	Shape Factor
I-section	1.14
Rectangle	1.5
Circular	1.697
Diamond	2
Triangle	2.34

Tubular Section	$\frac{4}{\pi}$

Load factor

Given by:$\frac{collapse\ load}{working\ load}$

→ depends upon shape of section

→ load factor = shape factor * FOS [fos = 1.5]

Method Of Analysis

1. statical method of analysis:

→ it is also called lower bound theorem.

→ plastic design of steel structure based on this method is safe.

2. Mechanism method or Kinematic method of analysis.

→ it is also called upper bound theorem

→ plastic design of steel structure based on this method is unsafe.

Length of elastic plastic hinge:

simply supported=$\frac{l}{3}$

I-section=$\frac{l}{5}$

Various Ratios

$$\frac{strength\ of\ fixed\ beam}{(strength\ of\ simply\ supported)}\ under\ \ UDl\ for\ shear\ =\ 1$$

$$\frac{strength\ of\ fixed\ beam}{(strength\ of\ simply\ supported)}\ under\ \ UDl\ for\ bending\ =\ 1.5$$

$$For\ strain\ energy\ stored\ in\ body, \frac{suddenly\ loaded}{gradually\ loaded}\ =\ 4$$

$$For\ stress\ induced\ in\ body, \frac{suddenly\ loaded}{gradually\ loaded}\ =\ 2$$

$$For\ member\ subjected\ to\ axial\ tensile\ load, \frac{greatest\ normal\ stress}{maximum\ shear\ stress} = 2$$

Maximum shear stress theory for failure of material at elastic limit is known as Guests or treca's theory.

Maximum principal stress theory for failure of material at elastic limit is known as Ranking's theory.

Maximum strain theory (बिचको same)→ known as st. venant's theory

total strain energy theory (बिचको same)→ Haig's theory

shear strain theory (बिचको same)→Von Mises